Energy associated with a static spherically symmetric nonsingular black hole

I-Ching Yang[†] [1] and Irina Radinschi[‡] [2]

[†]Department of Natural Science Education,
National Taitung Teachers College,
Taitung, Taiwan 950, Republic of China
and
[‡]Department of Physics, "Gh. Asachi" Technical University,
Iasi, 6600, Romania

ABSTRACT

We evaluate the energy distributions of the Dymnikova space-time using the Weinberg, Papapetrou, and Møller energy-momentum complexes. This result sustain the importance of the energy-momentum complexes in the evaluation of the energy distribution of a given space-time. To compare the energy distributions obtained by using several definitions, these results show that the Einstein, Tolman, and Weinberg energy complexes are the same in Schwarzschild Cartesian coordinates, but the Papapetrou and the Møller are not.

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[1]E-mail:icyang@dirac.phys.ncku.edu.tw
[2]E-mail:iradinsc@phys.tuiasi.ro

A large number of definitions of the gravitational energy according to General Relativity have been given. However, early energy-momentum investigations for gravitating system gave reference-frame-dependent energy-momentum complex [1, 2, 3, 4]; later the quasilocal idea [5] was developed. In their latest article, Chang, Nester and Chen [6] showed that the energy-momentum complexes are actually quasilocal and associated with the legitimate Hamiltonian boundary term. Whereas, several examples of particular space-times (the Kerr-Newman, the Einstein-Rosen, the Bonnor-Vaidya and the Kerr-Schild class) of black holes have been studied and different energy-momentum complexes are given the same energy distribution for a given space-time [7, 8, 9]. Recently, Yang [10] employing the Einstein energy-momentum complex obtained the energy distribution of the Dymnikova space-time that is positive everywhere and be equal to zero at origin. Later, Radinschi [11] calculate the energy distribution in the same one by using the Tolman energy-momentum complex and get the same result. In this article, we would like to evaluate the energy distributions of the Dymnikova space-time by using the Weinberg [2], Papapetrou [3] and Møller [4] energy-momentum complex.

In 1992, Dymnikova [12] obtained the static spherically symmetric non-singular black hole solution, which behaves like the Schwarzschild solution but with its singularity replaced by the de-Sitter core. The line element of solution is expressed as

$$ds^2 = \left(1 - \frac{R_g(r)}{r}\right) dt^2 - \left(1 - \frac{R_g(r)}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2, \quad (1)$$

where

$$R_g(r) = r_g \left(1 - \exp(\frac{r^3}{r_*^3})\right). \quad (2)$$

We have also $r_*^3 = r_g r_0^2$, $r_g = 2M$ and $r_0^2 = \frac{3}{8\pi\varepsilon_0}$. As $r \to \infty$, the line element of solution becomes the Schwarzschild solution, and as $r \to 0$, the line element of solution becomes the de Sitter solution with energy-momentum density $T_{\mu\nu} = \varepsilon_0 g_{\mu\nu}$.

First, let us consider the Weinberg energy-momentum complex [2]

$$Q^{\rho\nu\lambda} = \frac{1}{2}\left\{\frac{\partial h^\mu_\mu}{\partial x_\nu}\eta^{\rho\lambda} - \frac{\partial h^\mu_\mu}{\partial x_\rho}\eta^{\nu\lambda} - \frac{\partial h^{\mu\nu}}{\partial x_\mu}\eta^{\rho\lambda} + \frac{\partial h^{\mu\rho}}{\partial x_\mu}\eta^{\nu\lambda} + \frac{\partial h^{\nu\lambda}}{\partial x_\rho} - \frac{\partial h^{\rho\lambda}}{\partial x_\nu}\right\} \quad (3)$$

where the $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. The energy component of the Weinberg energy-momentum complex is most conveniently calculated in the quasi-Cartesian coordinate (t, x, y, z), and using Gauss's theorem gives

$$E_{\mathrm{W}}(r) = -\frac{1}{8\pi}\oint Q^{i00} n_i r^2 d\Omega \tag{4}$$

$$= -\frac{1}{16\pi}\oint (h_{ij,j} - h_{jj,i}) n_i r^2 d\Omega, \tag{5}$$

the integral begin taken over a large sphere of radius r, with $\vec{n}$ the outward normal and $d\Omega$ the differential solid angle; that is $n_i = x_i/r$, $r^2 = x_i x_i$, and $d\Omega = \sin\theta d\theta d\phi$. Herein the Latin index takes values from 1 to 3. In these coordinates, the line element (1) reads

$$h_{\mu\nu} = \begin{bmatrix} -R_g(r)/r & 0 & 0 & 0 \\ 0 & x^2\Psi/r^2 & xy\Psi/r^2 & xz\Psi/r^2 \\ 0 & xy\Psi/r^2 & y^2\Psi/r^2 & yz\Psi/r^2 \\ 0 & xz\Psi/r^2 & yz\Psi/r^2 & z^2\Psi/r^2 \end{bmatrix}, \tag{6}$$

where $\Psi = (1 - \frac{r}{r - R_g(r)})$. The required nonvanishing components Q^{i00} of the Weinberg energy complex are easily shown to be

$$Q^{100} = \frac{x}{r^2}(1 - \frac{r}{r - R_g(r)}), \tag{7}$$

$$Q^{200} = \frac{y}{r^2}(1 - \frac{r}{r - R_g(r)}), \tag{8}$$

$$Q^{300} = \frac{z}{r^2}(1 - \frac{r}{r - R_g(r)}). \tag{9}$$

Hence, the Weinberg energy complex within radius r reads

$$E_{\mathrm{W}}(r) = \frac{R_g(r)}{2}(1 - \frac{R_g(r)}{r})^{-1}. \tag{10}$$

Second, we calculate the energy component of the Papapetrou energy-momentum complex [3] in quasi-Cartesian coordinate

$$E_{\mathrm{P}}(r) = \frac{1}{16\pi}\int \frac{\partial B^{00i}}{\partial x^i} d^3x \tag{11}$$

with

$$B^{00i} = \frac{\partial}{\partial x^j}[\sqrt{-g}(g^{ij} + \eta^{ij} g^{00})]. \tag{12}$$

Herein the Latin index also takes values from 1 to 3. The required nonvanishing components of B^{00i} in the calculation of the Papapetrou energy complex are the following

$$B^{001} = \frac{x}{r^2}(R'_g(r) + \frac{R_g(r)}{r}) - \frac{x}{r}\left[\frac{rR'_g(r) - R_g(r)}{(r - R_g(r))^2}\right] \tag{13}$$

$$B^{002} = \frac{y}{r^2}(R'_g(r) + \frac{R_g(r)}{r}) - \frac{y}{r}\left[\frac{rR'_g(r) - R_g(r)}{(r - R_g(r))^2}\right] \tag{14}$$

$$B^{003} = \frac{z}{r^2}(R'_g(r) + \frac{R_g(r)}{r}) - \frac{z}{r}\left[\frac{rR'_g(r) - R_g(r)}{(r - R_g(r))^2}\right], \tag{15}$$

where $R'_g(r) = \frac{3r^2}{r_0^2}\exp(-\frac{r^3}{r_*^3})$. So, the energy within radius r in the Papapetrou prescription is given as

$$E_{\rm P}(r) = \frac{1}{4}(R_g(r) + rR'_g(r)) + \frac{1}{4}\left[\frac{R_g(r) - rR'_g(r)}{(1 - \frac{R_g(r)}{r})^2}\right]. \tag{16}$$

Finally, we evaluate the energy distribution by the Møller energy-momentum complex [4] in spherical coordinates

$$E_{\rm M}(r) = \frac{1}{8\pi}\int \frac{\partial \chi_0^{0i}}{\partial x^i} d^3x \tag{17}$$

with

$$\chi_0^{0i} = \sqrt{-g}g^{0\beta}g^{i\alpha}\left(\frac{\partial g_{0\alpha}}{\partial x^\beta} - \frac{\partial g_{0\beta}}{\partial x^\alpha}\right). \tag{18}$$

In which the Latin index also takes values from 1 to 3, but Greek indices run from 0 to 3. In his recent article about to analyze the Møller energy-momentum complex, Lessner [13] concludes that it is a powerful representation of energy and momentum in general relativity. Thus, Yang, Lin and Hsu [14] employing the Møller energy-momentum complex obtained the energy distribution of the charged dialton black hole recently. Later, Xulu [15] also computes the energy distribution in the Kerr-Newman space-time using the Møller energy-momentum complex. Notice the only nonvanishing component of Møller energy-momentum complex is

$$\chi_0^{01} = \sin\theta\left[r_g - r_g\exp(-\frac{r^3}{r_*^3}) - \frac{3r^3}{r_0^2}\exp(-\frac{r^3}{r_*^3})\right], \tag{19}$$

and the Møller energy-momentum complex within radius r is

$$E_{\rm M}(r) = M - M\exp(-\frac{r^3}{r_*^3}) - \frac{3r^3}{2r_0^2}\exp(-\frac{r^3}{r_*^3}). \tag{20}$$

In summary, the energy component of those three energy-momentum complexes reduce M as $r \to \infty$, corresponding to

$$E_{\mathrm{W}}(r)|_{r\to\infty} = E_{\mathrm{P}}(r)|_{r\to\infty} = E_{\mathrm{M}}(r)|_{r\to\infty} = M. \tag{21}$$

Therefore, the total energy is given by parameter M which is the same as the ADM mass for this metric, and is independent on definitions of energy-momentum complex. According to the Cooperstock hypothesis [16], the energy and momentum are confined to the region of non-vanishing energy-momentum tensor of matter and all non-gravitation fields. The standard formula for the mass of de Sitter-Schwarzschild solution obtained by Dymnikova

$$m(r) = \int_0^r T_0^0 d^3x = \frac{R_g(r)}{2} \tag{22}$$

is positive everywhere, unless at the origin, supports this hypothesis. About a decade age, Bondi [17] argued that a nonlocalizable form of energy is not allowed in relativity and therefore its location can in principle be found. Due to this reason, we try to comparision with the results of Yang [10]

$$E_{\mathrm{E}}(r) = \frac{r_g}{2}\left(1 - \exp\left(-\frac{r^3}{r_*^3}\right)\right) \tag{23}$$

and of Radinschi [11]

$$E_{\mathrm{T}}(r) = \frac{r_g}{2}\left(1 - \exp\left(-\frac{r^3}{r_*^3}\right)\right), \tag{24}$$

we could be a conclusion that

$$E_{\mathrm{E}}(r) = E_{\mathrm{T}}(r) = g_{00}E_{\mathrm{W}}(r) = \frac{R_g(r)}{2} \equiv E_{\mathrm{ETW}}(r) \tag{25}$$

while Dymnikova space-time is expressed by Schwarzschild Cartesian coordinates, but $E_{\mathrm{P}}(r)$ and $E_{\mathrm{M}}(r)$ are both not equal to $E_{\mathrm{ETW}}(r)$. In other words, these results should be support to the idea that the Einstein, Tolman and Weinberg energy-momentum complexes can give the same result for a given space-time. Also, this results sustain the idea [18] that the Papapetrou energy-momentum complex do not "coincide" with the Einstein, Tolman and Weinberg energy-momentum complexes in the Schwarzschild Cartesian coordinate.

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